سلم تصحيح امتحان مقرر رياضيات /4/

قسم الانتاج - فصل أول

/80/ الدرجة 2021 - 2022

السؤال الأول: (30 د)

1) $z^3 = \sqrt{3} - i \Rightarrow r = 2,\ \theta = -\frac{\pi}{6}$ (2)

$z = r^{\frac{1}{3}} e^{i\left(\frac{\theta + k2\pi}{3}\right)}$ (2)

$z_0 = 2^{\frac{1}{3}} e^{-i\frac{\pi}{18}},\quad z_1 = 2^{\frac{1}{3}} e^{i\left(\frac{11\pi}{18}\right)},\quad z_2 = 2^{\frac{1}{3}} e^{i\left(\frac{23\pi}{18}\right)}$ (6)

2) $f(x) = \sum_{n=-\infty}^{\infty} c_n e^{inx} \quad : \quad c_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} 3x\, e^{-inx}\, dx$

نكامل بالتجزئة (5)

$c_n = \frac{3}{2\pi}\int_{-\infty}^{\infty} x\, e^{-inx}\, dx$

$u = x \Rightarrow du = dx,\quad dv = e^{-inx} \Rightarrow v = -\frac{1}{in} e^{-inx}$

$c_n = \frac{3}{2\pi}\left\{ -\frac{x}{in} e^{-inx} \Big|_{-\pi}^{\pi} + \frac{1}{in}\int_{-\pi}^{\pi} e^{-inx}\, dx \right.$

$c_n = \frac{3}{2\pi}\left\{ \frac{-\pi}{in}(-1)^n - \frac{\pi}{in}(-1)^n \right\} + \frac{1}{n^2}\left\{ e^{-inx} \Big|_{-\pi}^{\pi} \right.$ (5)

$= \frac{3}{2\pi}(-1)^{n+1} \cdot \frac{\pi}{in} + 0$

$\Rightarrow f(x) = \sum_{-\infty}^{\infty} \frac{3\pi}{2\pi in}(-1)^{n+1} e^{inx}$

3) $f(z) = e^{-4(z+3)} = e^{-4(x-4iy+3)} = e^{-4x+3} \cdot e^{-4iy}$

$= e^{-4x+3}(\cos 4y - i \sin 4y)$ (6)

$u = e^{-4x+3}\cos 4y,\quad v = -e^{-4x+3}\sin 4y$

$$\frac{\partial u}{\partial x} = -4e^{-4x+3}\cos 4y, \quad \frac{\partial v}{\partial y} = -4e^{-4x+3}\cos 4y$$

$$\frac{\partial u}{\partial y} = -4e^{-4x+3}\sin 4y, \quad \frac{\partial v}{\partial x} = 4e^{-4x+3}\sin 4y$$

5

$$\Rightarrow \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \Rightarrow$$ تحليلي

السؤال الثاني (25 درجة)

$$I = \oint_C \frac{\operatorname{ch} z}{(z-2)(z-4i)}$$

$z_1 = 2, \; z_2 = 4i \quad : z_1 \in C, \; z_2 \notin C$

6

$$I = 2\pi i \sum \operatorname{Res}\{f, z_i\}$$

$$= 2\pi i \lim_{z \to 2} \frac{\operatorname{ch} z}{(z-2)(z-4i)}(z-2) = \frac{\operatorname{ch} 2i}{2-4i} \cdot 2\pi i$$

$z_1, z_2 \in C$

$$I = 2\pi i \left[\operatorname{Res}(f, 2) + \operatorname{Res}(f, 4i)\right]$$

$$= 2\pi i \left\{ \frac{\operatorname{ch} 2i}{2-4i} + \frac{\operatorname{ch} -4}{4i-2} \right\}$$

6

$$J = \int_{-\infty}^{\infty} \frac{\sin 4x}{x^2+25}\,dx$$

$$f(z) = \frac{1}{z^2+25} \longrightarrow 0$$

نسبي التكامل

$$J = \int \frac{e^{4iz}}{z^2+25}\,dz$$

$z^2+25 = 0 \Rightarrow z = \pm 5i$

نأخذ $z = 5i$ فوق المحور

5

$I = 2\pi i \, Res\{f\, e^{imz}, z_1\}$

(5) $2\pi i \, Res\{\frac{e^{4iz}}{z^2+25}, 5i\} = 2\pi i \lim_{z\to 5i} \frac{(z-5i)\, e^{4iz}}{(z-5i)(z)}$

$= \frac{\pi}{5} e^{-20}$

$\Rightarrow \int_{-\infty}^{\infty} \frac{\sin 4x}{x^2+25} = Im(\frac{\pi}{5} e^{-20}) = 0$ (3)

$\int_{-\infty}^{\infty} \frac{\cos 4x}{x^2+25} dx = Re(\frac{\pi}{5} e^{-20}) = \frac{\pi}{5} e^{-20}$

السؤال الثالث: (2 من 25)

1) z_0 قابلة للحذف $\Leftarrow$ (1) $Res\{f, z_0\} = 0$

z_0 قطب بسيط

$Res\{f, z_0\} \overset{(1)}{=} \lim_{z\to z_0} (z-z_0) f(z)$

3) z_0 قطب مرتبة ثانية (1)

$Res\{f, z_0\} = \frac{1}{1!} \lim \frac{d}{dz} (z-z_0)^2 f(z)$

$\lim_{z\to 0} \frac{\sin z}{z} = 1 \Rightarrow$ (2) z قابلة للحذف $\Rightarrow Res\{f\} = 0$

$\lim_{z\to 0} z \frac{\sin z}{z^2} \overset{(2)}{=} \lim \frac{\sin z}{z} = 1$

$Res\{f, 0\} = 1 \Leftarrow$ قطب بسيط $\Rightarrow z = 0$

$\int_0^{\infty} \frac{\cos 3t - \cos 4t}{t} dt = \int_0^{\infty} \phi(p) dp$

$= \int_0^{\infty} (\frac{p}{p^2+9} - \frac{p}{p^2+16}) dp$

$$= \left\{ \frac{1}{2}\ln(p^2+9) - \frac{1}{2}\ln(p^2+16) \right\}_0^{\infty} \quad 5$$

$$= \frac{1}{2}\ln\frac{p^2+9}{p^2+16}\Big]_0^{\infty} = \frac{1}{2}\left\{\ln 1 - \ln\frac{9}{16}\right\}$$

$$= \ln\frac{4}{3} \quad 4$$

3) $$\frac{3p-1}{(p-1)(p^2-4)} = \frac{A}{p-1} + \frac{B}{p-2} + \frac{C}{p+2}$$

$$\Rightarrow A = -\frac{2}{3},\ B = \frac{5}{4},\ C = -\frac{7}{12} \quad 6$$

$$L^{-1}\left\{\frac{3p-1}{(p-2)(p^2-4)}\right\} = -\frac{2}{3}e^{t} + \frac{5}{4}e^{2t} - \frac{7}{12}e^{-2t} \quad 3$$

مدرس المقرر

د. عدنان صيلي